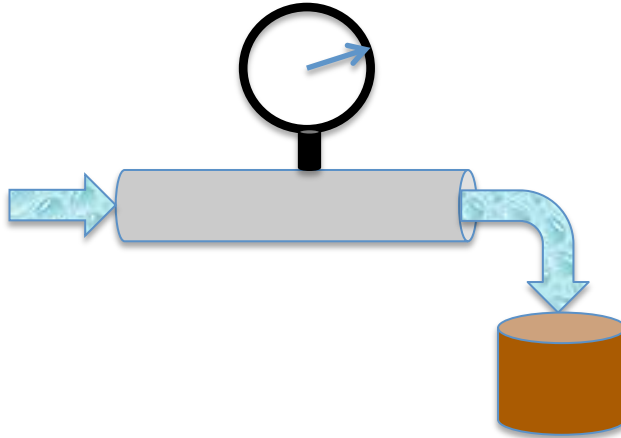


Mathematics Tutorial Series

The Fundamental Theorem of Calculus

Water in a pipe again!



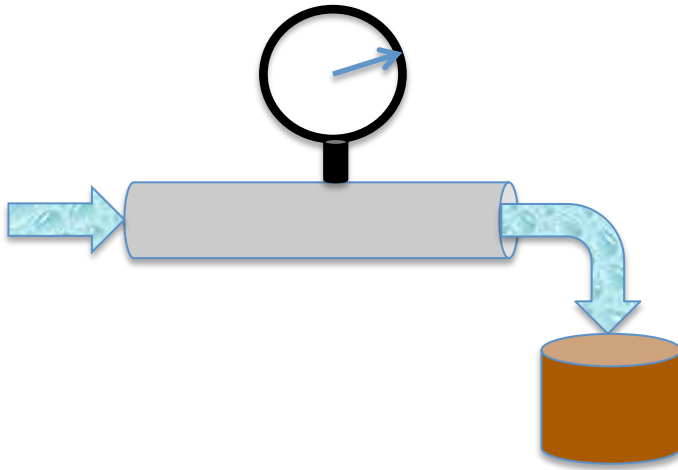
There are two ways to measure the total flow between times $t = a$ and $t = b$.

Our “perfect” approximation $\int_{t=a}^{t=b} f(t) dt$ is built from the rate of flow $f(t)$ by taking infinitely many measurement intervals of infinitesimal length. The “perfect” approximation is the limit of all those “imperfect” approximations.

So we can get the total flow from:

(1) a “perfect” approximation built from the flow rate $f(t)$:

$$\int_{t=a}^{t=b} f(t) dt$$



Or (2) we can just put a bucket under the spout!

Let $g(t)$ be the volume of water in the bucket at time t .

Then the volume of flow between times $t = a$ and $t = b$ is simple:

$$g(b) - g(a)$$

Then

(i) the flow rate is the rate of change $f(t)$ of the water in the bucket and

(ii) **The Fundamental Theorem of Calculus:**

if $f(t) = g'(t)$

$$\int_{t=a}^{t=b} f(t) dt = g(b) - g(a)$$